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Developments in Rare Kaon Decay Physics*

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ABSTRACT: We review the current status of the field of rare kaon decays. The study of rare kaon decays has played a key role in the development of the standard model, and the field continues to have significant impact. The two areas of greatest import are the search for physics beyond the standard model and the determination of fundamental standard-model parameters. Due to the exquisite sensitivity of rare kaon decay experiments, searches for new physics can probe very high mass scales. Studies of the $K \to \pi\nu\overline{\nu}$ modes in particular, where the first event has recently been seen, will permit tests of the standard-model picture of quark mixing and *CP* violation.

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1 INTRODUCTION

This article reviews the status of rare kaon decays, with emphasis on the progress made since the 1993 review in this series [1]. Several other excellent review articles are available focusing on rare kaon decays [2], theoretical studies of rare kaon decays [3-6], and non-rare kaon decays [7]. Due to limited space, we cannot cover many interesting topics, such as CP violation in $K_L^\circ \to \pi^+ \pi^-$ decays—ϵ and ϵ', or T/CPT violation in $K_L^\circ \to \pi^+ \pi^-$ decays, or searches for T violation in the transverse polarization of the μ^+ in $K^+ \to \pi^\circ \mu^+ \nu_\mu \gamma$ and $K^+ \to \mu^+ \nu_\mu \gamma$.

Kaons have a relatively long lifetime because they decay only through the weak interaction. As a result, studies of their decays provide key insights into the behavior of the weak interaction under the three fundamental symmetry operators C, P, and T. The first of these, C or charge conjugation, is a unitary operator that replaces particles by anti-particles. Thus, in one possible sign convention, $C|K^\circ\rangle = -|\overline{K}^\circ\rangle$ and $C|K^+\rangle = -|\overline{K}^-\rangle$. The parity operator, P, inverts spatial directions, replacing left by right and vice-versa. The kaons are pseudoscalar particles which are odd under the action of P. Under the combined operator CP, then

$$\begin{aligned} CP|K^\circ\rangle &= |\overline{K}^\circ\rangle \\ CP|\overline{K}^\circ\rangle &= |K^\circ\rangle \end{aligned} \tag{1}$$

Even and odd eigenstates of CP called K_1 and K_2 can then be formed from the symmetric and antisymmetric combinations of the K° and $\overline{K}^\circ$. If CP were an exact symmetry of the weak interaction, these combinations would be identified with the observed eigenstates of mass and lifetime, called K_S° and K_L°. The famous discovery in 1964 [8] of the decay $K_L^\circ \to \pi\pi$ implied that CP symmetry is violated in weak decays, since the K_L, which decays mostly to CP-odd final states, can also decay to $\pi\pi$, which is CP-even. We have since learned that nearly all of the $K_L \to \pi\pi$ decay can be explained by so-called *indirect* CP violation, in which the mass and lifetime eigenstates K_L and K_S are mixtures of the CP eigenstates given by

$$|K_S^\circ\rangle = \left(|K_1\rangle + \epsilon\,|K_2\rangle\right)/\sqrt{1+|\epsilon|^2} \tag{2}$$

$$|\overline{K}^{\circ}_{L}\rangle \quad = \left(|K_2\rangle + \epsilon\,|K_1\rangle\right)/\sqrt{1+|\epsilon|^2},$$

and the decay proceeds via $\epsilon\,|K_1\rangle$. A question that has been open until recently is whether there is also *direct* CP violation, in which the CP-odd eigenstate K_2 decays to CP-even final states such as $\pi\pi$. The traditional method of searching for this phenomenon, which is expected in the Standard Model, is to look for a small deviation from unity of the so-called double ratio

$$R = \frac{\Gamma(K_L \to \pi^\circ\pi^\circ)/\Gamma(K_S \to \pi^\circ\pi^\circ)}{\Gamma(K_L \to \pi^+\pi^-)/\Gamma(K_S \to \pi^+\pi^-)}. \tag{3}$$

The value of this ratio has been reported [9,10] to be significantly different from unity, establishing the existence of direct CP violation in weak interactions. The Standard Model predicts a variety of other direct-CP-violating effects in rare kaon decays; measurements of these processes, which are addressed in this article, can provide additional tests of the Standard Model picture of CP violation.

The anti-unitary operator T reverses the arrow of time. In Lorentz-invariant local field theories, like the Standard Model, the combined operator CPT is an exact symmetry of the Lagrangian. Thus the observed CP violation in kaon decays would imply the existence of T violation. However, it is also interesting to search for more direct evidence of T violation, and a number of kaon-decay experiments have also played a central role in this effort.

The field of rare kaon decays has a long and rich history: the discovery of the kaon in 1949 [11], the postulation of "strangeness" [12], the τ–θ puzzle [13] and the understanding of parity violation [14], the understanding of quark mixing [15,16], the discovery of CP violation [8], the small rate for $K^\circ_L \to \mu^+\mu^-$ and flavor-changing neutral currents (FCNCs) in general, and the development of the Glashow, Iliopoulos, Maiani (GIM) mechanism [17] and the prediction of the charm quark mass [18]. As the field has evolved, so has the definition of "rare" decays, from branching ratios of $\sim 10^{-3}$ to the current levels of $\sim 10^{-12}$.

This article will, in general, cover modes with branching ratios below $\sim 10^{-5}$, with one measured to be less than 10^{-11}. The two areas of greatest interest have been the very sensitive searches for physics beyond the standard model through lepton flavor-violating (LFV) decays and the studies of the standard-model picture of the Cabibbo, Kobayashi, Maskawa (CKM) mixing [16] and CP violation that have recently begun to bear fruit.

A large number of results from experiments at Brookhaven National Laboratory (BNL) (E787, E865, E871), Fermi National Accelerator Laboratory (FNAL) (E799-II: KTeV), and the European Laboratory for Particle Physics (CERN) (NA48) have been reported at recent conferences [19-32]. Many of these results have not yet been published.

1.1 Standard Model "Golden Modes" and CKM Matrix

The weak decay of quarks is described through the unitary CKM matrix. This matrix and the Wolfenstein parameterization [33,34] are shown below:

$$V_{\rm CKM} \quad = \quad \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{4}$$

$$\simeq \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\simeq \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{A^2\lambda^5(1-2\rho-2i\eta)}{2} & 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8}(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2[1-\lambda^2\frac{1-2\rho-i2\eta}{2}] & 1-\frac{A^2\lambda^4}{2} \end{pmatrix},$$

where λ is the sin of the Cabibbo angle, $\lambda \equiv \sin\theta_C \simeq 0.22$, and $\overline{\rho}$ and $\overline{\eta}$ are related to the Wolfenstein parameters ρ and η by $\overline{\rho} \equiv \rho(1-\lambda^2/2)$ and $\overline{\eta} \equiv \eta(1-\lambda^2/2)$.

The unitarity of this matrix can be expressed in terms of six unitarity conditions which can be represented graphically in the form of triangles, all of which have the same area. The area of these triangles is equal to one half of the Jarlskog invariant, J_{CP} [35]. This is the fundamental measure of *CP* violation in the standard model. One of the possible unitarity relations that is frequently cited in the literature is

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \tag{5}$$

This equation can be represented graphically, as in Figure 1, where we have divided all sides by $V_{cb}^* V_{cd}$, which is a real quantity to $\mathcal{O}(\lambda^6)$. This particular representation provides a convenient display, with the apex of the triangle given by the two least well-known of the Wolfenstein parameters, $\overline{\rho}$ and $\overline{\eta}$. The best information currently comes from several measurements of B meson decays, as well as the measured value of ϵ from kaon decays. All of the unitarity triangles should be tested; it is desirable to overconstrain each of the unitarity relations and to measure J_{CP} in each of the triangles.

The most powerful tests of our understanding of *CP*-violation and quark mixing will come from comparison of the results from B meson and kaon decays with little theoretical ambiguity. The two premier tests are expected to be:

- Comparison of the angle β from the ratio B($K_L^\circ \to \pi^\circ \nu\overline{\nu}$)/B($K^+ \to \pi^+ \nu\overline{\nu}$) and the *CP*-violating asymmetry in the decay $B_d^\circ \to \psi K_S^\circ$.
- Comparison of the magnitude $|V_{td}|$ from $K^+ \to \pi^+ \nu\overline{\nu}$ and the ratio of the mixing frequencies of B_s to B_d mesons, expressed in terms of the mass difference ratio $\Delta M_{B_s}/\Delta M_{B_d}$.

The current value of the fundamental level of *CP* violation in the SM, $J_{CP} = (2.7 \pm 1.1) \times 10^{-5}$, is known, primarily from measurements of B meson decays, with about 40% uncertainty [36]. Measurement of J_{CP} in the kaon system is very clean theoretically (uncertainty of ~2%) and can be expected to be measured to ~8% within a decade. While measurement of J_{CP} in the B system is difficult and is plagued by theoretical uncertainties, it is likely that a 15% measurement is possible and if this could be pushed to the level expected from the kaon system, the comparison of these values will also be an important test of the SM.

1.2 Form Factor Measurements

Interest in rare kaon decays extends well beyond their potential to determine standard-model parameters. Dozens of different medium-rare (branching ratios in the range 10^{-5} to 10^{-8}) kaon decays have been measured. With the ever-increasing sensitivity of experiments designed to search for the very-rare modes

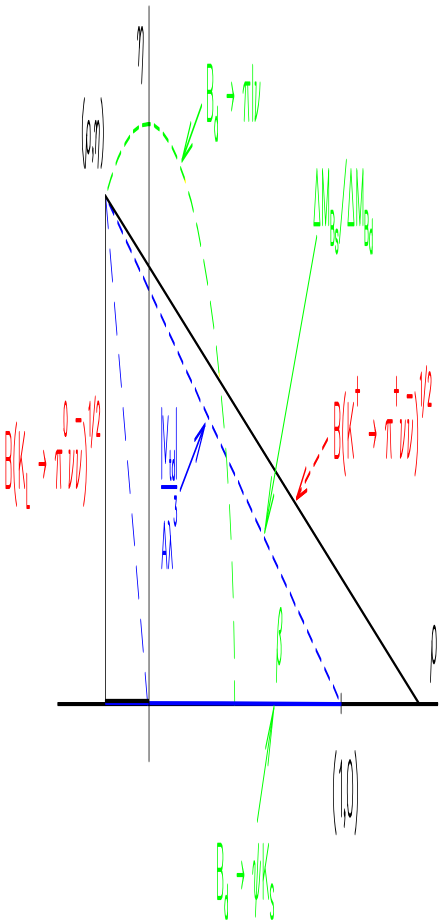

Figure 1: Traditional representation of the unitarity triangle. Measurements of B meson decays introduce constraints shown in green, contributions from the two golden kaon decay modes are marked in red.

that probe standard-model parameters or search for new physics, the statistics available for these medium-rare decays have increased to the point where both precision branching ratio measurements and form factor studies are possible.

Both the branching ratios and the form factors provide excellent tests of chiral perturbation theory (ChPT) [37,38], which should work well at the relatively low momentum scales characteristic of kaon decays. The wide variety of different modes and form factors can be used to test ChPT.

Measurements of a number of modes, such as $K^\circ_L \to e^+e^-\gamma\gamma$ and $K^\circ_L \to \pi^\circ\gamma\gamma$, are directly relevant to the determination of standard-model parameters because these modes can be backgrounds to more interesting decays, such as $K^\circ_L \to \pi^\circ e^+e^-$ or $K^\circ_L \to \pi^\circ\nu\overline{\nu}$. They can also provide information necessary to disentangle different amplitudes contributing to the signal mode, such as the $\pi^\circ\gamma^*\gamma^*$ intermediate state for $K^\circ_L \to \pi^\circ e^+e^-$ or the $\gamma^*\gamma^*$ intermediate state for $K^\circ_L \to \mu^+\mu^-$.

The study of these "non-marquee" decay modes is thus more than a beneficial by-product of experiments designed to search for the more significant decays. Their properties are often of vital importance to the determination of backgrounds, the extraction of standard-model parameters, or tests of the reliability of theoretical tools like ChPT.

1.3 Searches for New Physics

A major thread in the history of the study of rare kaon decays is the search for exotic phenomena, often referred to as "beyond the standard model" (BSM). The quintessential example is the long search for the decay $K^\circ_L \to \mu e$. This decay is absolutely forbidden in the standard model with massless neutrinos; specifically, it is forbidden by the symmetry of conserved lepton flavor number, for which no fundamental reason is known. Grand unified theories or other extensions to the standard model often contain heavy vector bosons that connect the standard-model lepton families—for example, coupling muons to electrons (horizontal gauge bosons) or quarks to leptons (leptoquarks). Both types of particles could mediate the otherwise forbidden decays, such as $K^\circ_L \to \mu e$ or $K \to \pi\mu e$.

Because these decays simply do not happen in the standard model and are relatively simple to detect, they provide exceptional sensitivity to BSM physics. With experiments now probing branching ratios at the level of 10^{-12}, even a very heavy exotic boson (on the order of 100 TeV, for the usual electroweak coupling) would lead to a detectable signal.

2 *CP* VIOLATION AND THE CKM MATRIX

The unitarity triangle is most readily expressed for the kaon system as follows:

$$\begin{aligned} V^*_{us}V_{ud} + V^*_{cs}V_{cd} + V^*_{ts}V_{td} &= 0 \qquad (6) \\ \text{or} & \\ \lambda_u + \lambda_c + \lambda_t &= 0, \end{aligned}$$

with the three vectors $\lambda_i \equiv V^*_{is}V_{id}$ converging to form a very elongated triangle in the complex plane. This is illustrated graphically in Figure 2. The first vector, $\lambda_u = V^*_{us}V_{ud}$, is well known. The height will be measured by $K^\circ_L \to \pi^\circ\nu\overline{\nu}$ and the third vector, $\lambda_t = V^*_{ts}V_{td}$, will be measured by the decay $K^+ \to \pi^+\nu\overline{\nu}$. The theoretical ambiguities in interpreting all of these measurements are very small.

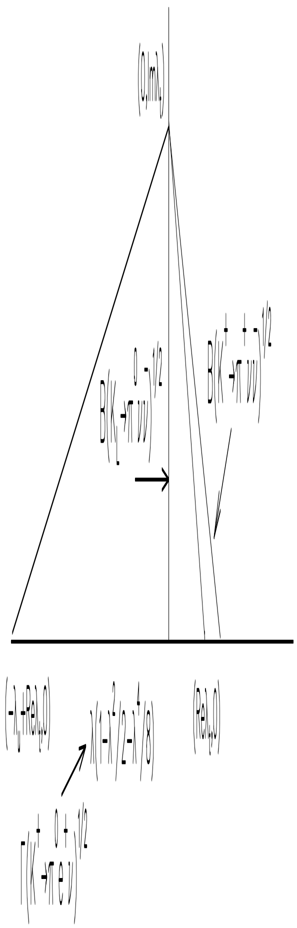

Figure 2: Unitarity triangle for the K system (not to scale).

It may be possible to extract additional constraints on the height of the triangle from $K_L^\circ \to \pi^\circ \ell^+ \ell^-$ decays and on $Re(\lambda_t)$ from $K_L^\circ \to \mu^+\mu^-$ decays.

The base of this triangle has the length $b \equiv \lambda_u = V_{us}^* V_{ud}$, determined from the decay rate of $K \to \pi e \nu_e$ and nuclear beta decay. If we assume unitarity then b is determined completely from $K \to \pi e \nu_e$ and $b = |V_{us}|$ to very good approximation. To even better accuracy it is expressed as

$$b = \lambda - \frac{\lambda^3}{2} - \frac{\lambda^5}{8}. \tag{7}$$

The value of λ, the best-known of the Wolfenstein parameters, is extracted [39] from the measurement of the $K \to \pi e \nu_e$ rate [40]. The height of the triangle, $h \equiv Im(\lambda_t)$ can be derived from a measurement of the $K_L^\circ \to \pi^\circ \nu \overline{\nu}$ branching ratio. The area of the triangle, a, is then given by two kaon decay measurements as

$$J_{CP} = 2a = b \times h = \lambda_u \times Im(\lambda_t) = 0.976 \times \lambda \times Im(\lambda_t); \tag{8}$$

the ultimate uncertainty on $Im(\lambda_t)$ and a will be limited, not by theoretical ambiguities, but by experimental uncertainties on $\mathrm{B}(K_L^\circ \to \pi^\circ \nu \overline{\nu}\ )$, to $\mathcal{O}(5-10\%)$ from the next round of $K_L^\circ \to \pi^\circ \nu \overline{\nu}$ experiments. This compares favorably to the B system, where three (four without the unitarity assumption) measurements are needed.

Table 1 lists current values [40-42] for the magnitudes of the CKM matrix elements and Wolfenstein parameters.

Table 1: Magnitudes of CKM matrix parameters. The current values for the matrix elements V_{ji} are listed, where i loops over the d-type and j represents the u-type quarks, as are the $\lambda_j \equiv V_{js}^* V_{jd}$ values as defined earlier in the text and the Wolfenstein parameters (λ, A, ρ and η).

V_{ji}	i=d	i=s	i=b	$\lambda_j \equiv V_{js}^* V_{jd}$
V_{ui}	0.9740 ± 0.0010	0.2196 ± 0.0023	0.0032 ± 0.0008	0.2139 ± 0.0026
V_{ci}	0.224 ± 0.016	1.04 ± 0.16	0.0395 ± 0.0017	0.233 ± 0.040
V_{ti}	0.0084 ± 0.0018[1]	$\sim V_{cb}$ [1]	0.99 ± 0.29	$.00033 \pm .00009$
λ	0.2196 ± 0.0023 [40]			
A	0.819 ± 0.039 [40]			
ρ	0.14 ± 0.15 [41] $(0.18 \pm 0.04$ [42]$)$[2]			
η	0.38 ± 0.13 [41] $(0.36 \pm 0.03$ [42]$)$[2]			

[a] The entries for V_{td} and V_{ts} assume a 3 generation unitary matrix.

[b] The uncertainties on ρ and η are conservative, generally accepted values. Values from more aggressive treatment of errors are given in parentheses.

2.1 $K_L^\circ \to \mu^+\mu^-$

The decay $K_L^\circ \to \mu^+\mu^-$ is dominated by the process of $K_L^\circ \to \gamma\gamma$ with the two real photons converting to a $\mu^+\mu^-$ pair. This contribution can be precisely calculated in QED [43] based on a measurement of the $K_L^\circ \to \gamma\gamma$ branching ratio. However, there is also a long-distance dispersive contribution, through off-shell photons. This contribution needs additional input from ChPT [44,45], which may be aided by new, improved measurements of the decays $K_L^\circ\to e^+e^-\gamma$, $K_L^\circ\to\mu^+\mu^-\gamma$, $K_L^\circ\to e^+e^-e^+e^-$ and $K_L^\circ\to\mu^+\mu^-e^+e^-$ (see Section 3.1), although there is some dispute as to the reliability of such calculations [46,47]. Most interesting is the short-distance contribution which proceeds through internal quark loops, dominated by the top quark (see Figure 3). This contribution is sensitive to

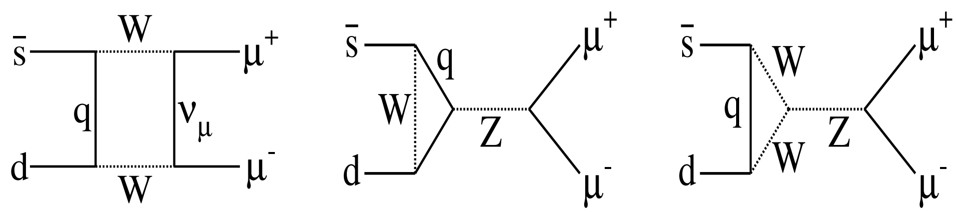

Figure 3: Feynman diagrams for the short-distance component of the decay $K_L^\circ\to\mu^+\mu^-$.

the real part of the poorly known CKM matrix element V_{td} or equivalently to ρ [5,48]. If this were the only contribution to the decay, the branching ratio $B_{SD}(K_L^\circ\to\mu^+\mu^-)$ could be written as

$$\begin{aligned} B_{SD}(K_L^\circ\to\mu^+\mu^-) &= \frac{\tau_L}{\tau_{K^+}}\frac{\alpha^2 B(K_{\mu 2})}{\pi^2\sin^4\theta_W|V_{us}|^2}[Y_c Re(\lambda_c)+Y_t Re(\lambda_t)]^2 \\ &= 1.51\times 10^{-9}A^4(\rho_0-\overline{\rho})^2, \end{aligned} \tag{9}$$

with $\rho_0 = 1.2$ and the Inami-Lim functions [5,49], Y_q, are functions of $x_q \equiv M_q^2/M_W^2$ where M_W is the mass of the W boson and M_q is the mass of the quark q. This mode has now been measured with impressively high statistics [50] (see Figure 4) by the E871 collaboration (see Section 6.1.2). The branching ratio, B($K_L^\circ\to\mu^+\mu^-$) $= (7.18\pm 0.17)\times 10^{-9}$, is a factor of three more precise than previous measurements, and the error on the rate relative to $K_L^\circ\to\pi^+\pi^-$,

$$\frac{\Gamma(K_L^\circ\to\mu^+\mu^-)}{\Gamma(K_L^\circ\to\pi^+\pi^-)} = (3.474\pm 0.054)\times 10^{-6}, \tag{10}$$

no longer dominates the error on the ratio

$$\frac{\Gamma(K_L^\circ\to\mu^+\mu^-)}{\Gamma(K_L^\circ\to\gamma\gamma)} = (1.213\pm 0.030)\times 10^{-5}, \tag{11}$$

contributing only $\sim$1.5% of the 2.5% error. The remaining significant sources of uncertainty,

$$\frac{\Gamma(K_L^\circ\to\gamma\gamma)}{\Gamma(K_L^\circ\to\pi^\circ\pi^\circ)} = 0.632\pm 0.009\ , \qquad \frac{\Gamma(K_S^\circ\to\pi^+\pi^-)}{\Gamma(K_S^\circ\to\pi^\circ\pi^\circ)} = 2.186\pm 0.028, \tag{12}$$

will probably be improved in the near future by the KLOE experiment at Frascati.

This measured ratio is only slightly above the unitarity bound from the on-shell two-photon contribution

$$\frac{\Gamma(K_L^\circ\to\mu^+\mu^-)}{\Gamma(K_L^\circ\to\gamma\gamma)} = 1.195\times 10^{-5} \tag{13}$$

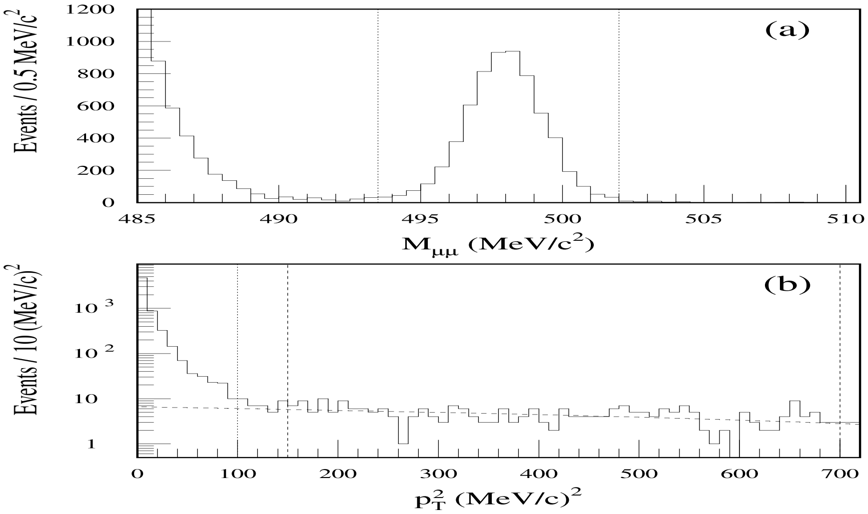

Figure 4: Final sample of $K_L^\circ \to \mu^+\mu^-$ decays from experiment E871 at BNL. A total of ~6200 $K_L^\circ \to \mu^+\mu^-$ events are observed in the peak. a) Reconstructed mass of the $\mu^+\mu^-$ pair, $M_{\mu\mu}$ and b) the momentum of the reconstructed $\mu^+\mu^-$ pair relative to that of the parent kaon (p_T), where the direction of the parent kaon is derived from the locations of the target and the decay vertex.

and limits possible short-distance contributions. With a recent estimate of the long-distance dispersive contribution [44], a limit on ρ was extracted: $\rho > -0.33$ at 90% confidence level (CL) [50].

Unlike $K_L^\circ \to \mu^+\mu^-$, which is predominantly mediated by two real photons, the decay $K_L^\circ \to e^+e^-$ proceeds primarily via two off-shell photons. The relative contribution from short-distance top loops is significantly smaller than in $K_L^\circ \to \mu^+\mu^-$. However, the recent observation by E871 [51] of four events, with a branching ratio of $B(K_L^\circ \to e^+e^-\ ) = (8.7^{+5.7}_{-4.1}) \times 10^{-12}$, is consistent with ChPT predictions [45,46] and is the smallest branching ratio ever measured for any elementary particle decay.

2.2 $K_L^\circ \to \pi^\circ \ell^+ \ell^-$

The decays $K_L^\circ \to \pi^\circ e^+e^-$ and $K_L^\circ \to \pi^\circ \mu^+\mu^-$ can proceed via the direct-CP-violating components of the diagrams in Figure 3 and the $s \to d\gamma^*$ amplitude, where γ^* represents a virtual photon. These processes are calculable with high precision within the Standard Model since they are dominated by top-quark exchange. If these were the only contributions to this decay, the branching ratio for $K_L^\circ \to \pi^\circ e^+e^-$ would be related to the CKM matrix elements by [52]

$$\begin{aligned} B_{SD}(K_L^\circ \to \pi^\circ e^+e^-\ ) &= \frac{\tau_L \alpha^2 B(K_{e3})}{\tau_{K^+} 4\pi^2 |V_{us}|^2}(y_{7A}^2 + y_{7V}^2)|Im(\lambda_t)|^2 \\ &= 6.91 \times 10^{-11} A^4 \eta^2, \end{aligned} \tag{14}$$

With the current best-fit value of 1.38×10^{-4} for $Im(\lambda_t)$ [53], this implies a branching ratio of about 5×10^{-12}. The related decay $K_L^\circ \to \pi^\circ \mu^+\mu^-$ is expected to be suppressed relative to the electron mode by about a factor of five owing to the reduced phase space.

Unfortunately, the decay $K_L^\circ \to \pi^\circ e^+ e^-$ can occur in two other ways. First, there is an indirect-*CP*-violating contribution from the *CP*-even, K_1° component of K_L°. This contribution could be determined from a measurement of $K_S^\circ \to \pi^\circ e^+ e^-$, but the current upper limit [54] of $B(K_S^\circ \to \pi^\circ e^+ e^-) < 1.6 \times 10^{-7}$ is far from the expected level of less than 10^{-8}. A K_S branching ratio of 10^{-9} would imply an indirect-*CP*-violating contribution to $K_L^\circ \to \pi^\circ e^+ e^-$ of about 3×10^{-12}, comparable to the expected direct-*CP*-violating contribution. In addition, a significant contribution is expected from the interference between the direct and indirect-*CP*-violating amplitudes.

Further complicating the picture is the presence of a *CP*-conserving amplitude involving a $\pi^\circ \gamma^* \gamma^*$ intermediate state, from which the virtual photons materialize into an e^+e^- pair. A model for the $K_L^\circ \pi^\circ \gamma^* \gamma^*$ vertex is needed to determine the size of this contribution. This vertex can be studied by measuring the $K_L^\circ \to \pi^\circ \gamma\gamma$ decay and the related decay $K_L^\circ \to \pi^\circ e^+ e^- \gamma$. These decays are discussed in Section 3.2. Based on recent measurements of these modes, by the KTeV experiment at Fermilab (see Section 6.2.1), the *CP*-conserving contribution to $K_L^\circ \to \pi^\circ e^+ e^-$ has been estimated at 1–2$\times 10^{-12}$, comparable to the expected direct-*CP*-violating part.

Given these three contributions, it will be difficult to extract CKM matrix parameters from even a precision measurement of $K_L^\circ \to \pi^\circ e^+ e^-$. But there is a still more formidable roadblock to progress on these modes, first pointed out by Greenlee [55]. As is discussed in Section 3.1, the radiative Dalitz decay $K_L^\circ \to e^+ e^- \gamma\gamma$ has a rather large branching ratio ($\sim 6 \times 10^{-7}$). The two photons may have an invariant mass near that of the π°, so that the final state is indistinguishable from the $\pi^\circ e^+ e^-$ mode. Two strategies can be used to deal with this background. First, a high-precision calorimeter can be used to minimize the size of the region in $M_{\gamma\gamma}$ where confusion can occur; second, the difference in the kinematic distributions expected in the radiative Dalitz decay can be used to remove most of the background events, at a cost of some acceptance for the signal mode $K_L^\circ \to \pi^\circ e^+ e^-$. These techniques reduce, but cannot eliminate, this background, so that the present searches for $K_L^\circ \to \pi^\circ e^+ e^-$ are background-limited at the level of 10^{-10}.

The most recent limit on $K_L^\circ \to \pi^\circ e^+ e^-$ comes from the KTeV experiment [56]. The analysis selects on the direction of the photons with respect to the electrons to minimize the background from radiative Dalitz decays while preserving as much sensitivity as possible. Figure 5*a* shows the $ee\gamma\gamma$ mass vs the $\gamma\gamma$ mass for the KTeV data with the signal box excluded, and Figure 5*b* shows an expanded view around the signal box. KTeV found two events that passed all cuts, compared with an expected background level of 1.06 events. This finding leads to an upper limit $B(K_L^\circ \to \pi^\circ e^+ e^-) < 5.6 \times 10^{-10}$ (90% CL). Although this limit represents a significant improvement over previous results [57-59], it is still two orders of magnitude above the standard-model prediction for the direct-*CP*-violating component of this decay.

A similar analysis of the related muon mode by KTeV resulted in a slightly smaller upper limit [60], $B(K_L^\circ \to \pi^\circ \mu^+ \mu^-) < 3.4 \times 10^{-10}$ (90% CL). Figure 6 shows a plot of the $\pi^\circ \mu\mu$ mass, with two events in the signal region and an expected background of 0.87 ± 0.15. The greater sensitivity for this mode results from making looser kinematic cuts. Due to the reduced phase space for this decay, the branching ratio of the muon mode is expected to be a factor of five smaller than that for the electron mode, so that this limit is farther from the expected